Faster, better, cheaper: News on seeking Gaia's Astrometric Solution with AGIS

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Abstract

Gaia is ESA's ambitious space astrometry mission with a foreseen launch date in early 2012. Its main objective is to perform a stellar census of the 1000 Million brightest objects in our galaxy (completeness to V=20 mag) from which an astrometric catalog of micro-arcsec level accuracy will be constructed. A key element in this endeavor is the Astrometric Global Iterative Solution (AGIS) - the mathematical and numerical framework for combining the ~80 available observations per star obtained during Gaia's 5yr lifetime into a single global astrometric solution. At last year's ADASS XVIII we presented (O4.1) in detail the fundamental working principles of AGIS, its development status, and selected results obtained by running the system on processing hardware at ESAC, Madrid with large-scale simulated data sets. We present here the latest developments around AGIS highlighting in particular a much improved algebraic solving method that has recently been implemented. This Conjugate Gradient scheme improves the convergence behavior in significant ways and leads to a solution of much higher scientific quality.

AGIS in a nutshell

- AGIS is the mathematical and numerical scheme that shall be used to generate the core astrometric mission products of Gaia: The five (six) standard astrometric parameters, position (2x), parallax (1x), proper motion (2x), radial velocity (1x) for all observed celestial objects brighter than G=20mag with targeted accuracies (e.g. <10 uas [G<10mag], <25 uas [G=15mag], <300uas [G=20]).
- The best global match between all measurement data and an observational model involving the unknown source, attitude, calibration, and global parameters is sought in a least-square sense using normal equations and a block-iterative solution scheme
- The system is developed in DPAC's CU3 by ESAC (Spain) and Lund Observatory (Sweden)
- AGIS shall run every 6 months on an ever increasing raw data volume
- The operational system is baselined to run on dedicated processing hardware at ESAC – a 10 TFLOP/s machine is envisaged – an alternative "Cloud" computing solution is being studied

- Continuous large-scale validation tests aim at
  - Demonstrating that the targeted mission accuracies can be reached
  - Study convergence behavior of the system and devise acceleration schemes
- About 40 iterations are currently (with SI scheme, see below) needed to converge to a solution starting from conservative, noisy initial starting values

Algebraic Details

- Assuming a centered Gaussian observation noise of standard deviation s*L* the astrometric problem is equivalent to a weighted least-square minimization of

$$\min_{\boldsymbol{x}} \; J(\mathbf{x}) = \underbrace{\sum_L}_{\text{sum over observations}} \left(t_L^{\mathrm{obs}} - f_L(\mathbf{x}, \mathrm{aux})\right)^2 \underbrace{\frac{w_L}{\sigma_L^2 + \varepsilon_L^2}}_{\text{statistical weight}} = \sum_L R_L(\mathbf{x}, \mathrm{aux})^2 W_L$$

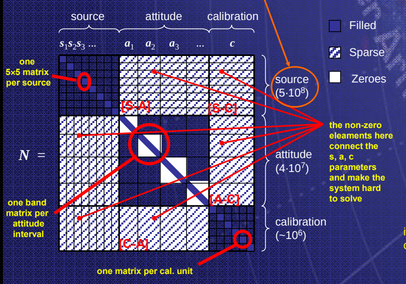
Observed-Calculated (Residual)

- So, the solution vector **x**=(s_i, a_j, c_k) gives the optimal agreement between the observational model *f* and the actual observations
- What is really solved in practice is a system of normal equations system for the updates **Δ*x*** to the parameters ***x***:

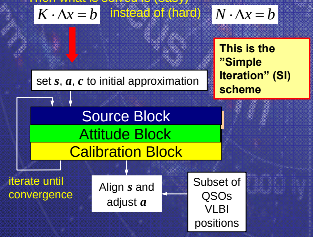
$$\left[\sum_L \frac{\partial R_L}{\partial \mathbf{x}} \frac{\partial R_L}{\partial \mathbf{x}'} W_L\right] \cdot \Delta\mathbf{x} = -\sum_L \frac{\partial R_L}{\partial \mathbf{x}} R_L W_L \quad \Leftrightarrow \quad \mathbf{N} \cdot \Delta\mathbf{x} = \mathbf{b}$$

N is a symmetric *n*×*n* matrix, ***b*** is an array of length *n* = number of parameters

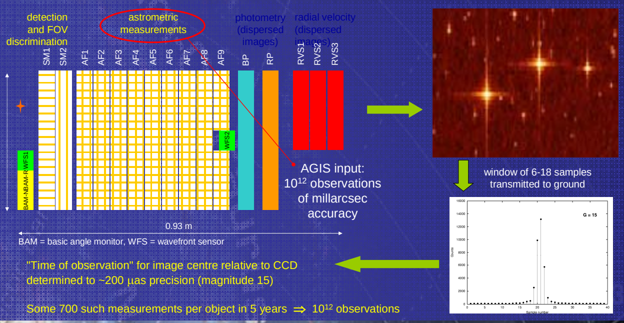
Most of AGIS is about solving the system $N \cdot \Delta x = b$
This needs to be done by iterations because of non-linearities and the very large size of the Normal equation matrix N



- Setting the "disturbing" off-diagonal blocks [S-A], [S-C], [A-C], [C-A] to **zero** gives an approximation to *N* which is the Gauss-Seidel Preconditioner matrix *K*
- Then what is solved is (easy) $K \cdot \Delta x = b$ instead of (hard) $N \cdot \Delta x = b$

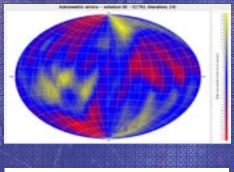


AGIS input: Gaia's astrometric measurements



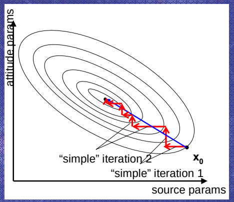
"Simple Iterations" and "Conjugate Gradients"

- Simple Iterations works but converges very slowly – after about 40 iterations the spatial errors show still "zonal patterns"



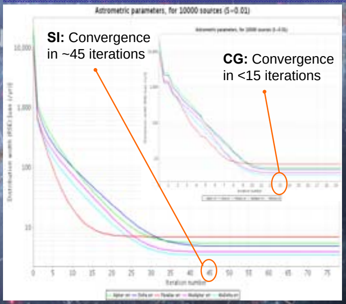
Mean error about 20 muas max, so the system has reduced the initial error by more than 3 powers of 10

Error pattern shows the eigenvector corresponding to the largest eigenvalue of the iteration matrix (I – K⁻¹N). Solution is NOT converged!



- A problem with SI is that updates within a block are constrained to be along the data space axes and there is no direct "cross-talk" between the blocks
- Conjugate Gradients (CG) is a standard linear algebra method to solve large systems of linear equations - CG does not have this limitation – in theory it could reach the minimum in a single step

Results



- ➢So far only results from AGISLab:
  - ❖CG works and can produce a really converged solution, free of systematic errors
  - ❖Number of needed iterations is reduced by a factor **2-3**
- ➢Implementation in AGIS completed – validation is ongoing – preliminary results are encouraging
- ➢Convergence acceleration perhaps less spectacular than in AGISLab because of outliers in data (AGISLab uses ideal, noisefree data)

AGIS and Nano-JASMINE

- Nano-Jasmine is a small Japanese near-infrared (z-band: 0.6-1 µm) astrometry mission - foreseen launch: late 2010 - lifetime: 2 years
- Aims at completeness to z=10 (~1 Mio stars) - expected accuracy: 2-3 mas (roughly "Hipparcos")
- Design and operating principles very similar to Gaia (2 telescopes separated by basic angle, satellite is spinning about its main symmetry axis, one single CCD in TDI mode)
- **Aim is to construct the astrometric catalog with AGIS**
- No "special" AGIS version required – only needs nano-JASMINE version of Initial Data Treatment (IDT) and the exchange of Gaia Parameter Data Base by Nano-Jasmine version
- AGIS should process Nano-Jasmine data like coming from a "crippled" Gaia with only 1 CCD strip and row