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An algorithm of refinement of image alignment for image subtraction

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Abstract. An algorithm is presented for estimating a shift between a pair of images. When subtracting a image from another image with a slight shift, we recognize many positive-negative (bipolar) pattern around objects. The idea of this study is to use the pattern for estimating the best alignment solution. Moving one image around the other, we can draw many vectors toward the best position, and estimate the best position as the intersection of the vectors.

1. Introduction

It is required to take a difference of two images in some astronomical data analysis. One example is a narrow-line imaging; subtracting an off-band image from an on-band image to see a morphology of emission line regions. Another example is a search for moving objects; take a difference of different epochs to exclude static objects. In the image subtraction, several preprocesses are important, such as a point spread function (PSF) matching (e.g. Alard&Lupton 1998), a flux scale matching, and a position alignment. If PSF size is mismatched, point sources in the difference image will have ring-shape. If flux scale is mismatched, most of objects in the difference image are all positive/all negative, and target features disappear. And if a slight positional shift exists, positive-negative(bipolar) patterns appear in the difference image.

Here I will present a method to estimate the relative positional shift between two images using the positive-negative patterns, so that we can make a better difference image. The method requires that the input images have similar PSF and similar flux scale, because the positive-negative pattern does not appear if the two input images have very different PSF or very different flux scale.

Hereafter, in subtraction of images, $C = A - B$, I will call A, B, and C as the target image, the reference image, and the difference image, respectively. The problem is how to align the target image(A) with the reference image(B).

2. Basic Idea

Dealing with the difference images, I found that the distance between the positive peak and the negative peak is almost independent of the amount of the shift. An simple example of one dimensional gaussian case is shown as follows. Assume

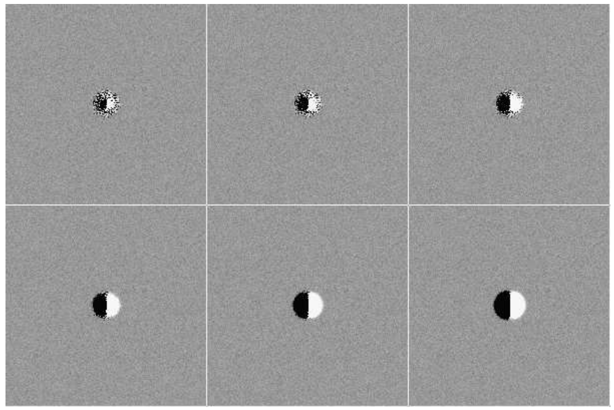

Figure 1. The difference of two PSFs with a small relative shifts. The PSF is a double gaussian of FWHM=5 pixels. The relative shifts of x direction are 0.001, 0.003, 0.01, 0.03, 0.1 and 0.3 pixels from left to right and from top to bottom, respectively.

that the profile of an object follows a gaussian;

$$g(x) = exp\left[-\frac{x^2}{2\sigma^2}\right].$$

If the shift between the target and the reference is $2s$, the profile in the difference image is

$$f(x) = exp\left[-\frac{(x-s)^2}{2\sigma^2}\right] - exp\left[-\frac{(x+s)^2}{2\sigma^2}\right]$$

The peak position of the profile is obtained by solving f'(x)=0. The equation is simplified as

$$-(x-s)exp\left[\frac{sx}{\sigma^2}\right] + (x+s)exp\left[\frac{-sx}{\sigma^2}\right] = 0.$$

If $sx << \sigma^2$, we can approximate that $exp\left[\frac{sx}{\sigma^2}\right] \sim 1 + \frac{sx}{\sigma^2}$. Substituting the approximation to the equation, the solution x=$\pm\sigma$ is obtained. In the case, $sx << \sigma^2$ is equivalent to $s << \sigma$. From numerical calculation, the exact solutions are $x = \pm 1.04\sigma$ and $x = \pm 1.12\sigma$ for $s = 0.5\sigma$, and $s = \sigma$, respectively. It means that if the relative shift s is smaller than σ, the distance between the positive and the negative peaks in the difference image is $\sim 2\sigma$, regardless of the amount of the shift s.

In Figure 1, 2D simulated difference images of slightly shifted PSFs are shown. The figure clearly shows that the change of shift does not affect the position of the peaks. Meanwhile, the change of shift affects the S/N of positive-negative pattern.

3. Method

Though the positive-negative pattern loses the information of the amount of the shift, the pattern holds the information of the direction of the shift. In Figure

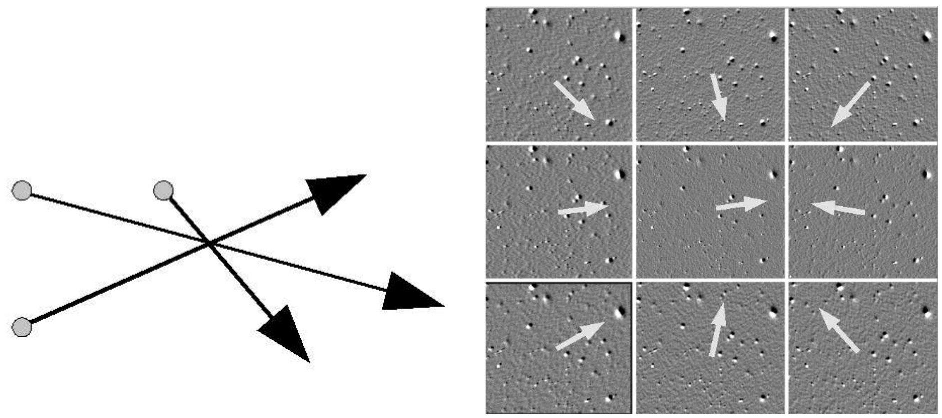

Figure 2. (left) Schematic figure of the method. If we know the direction of the shift at two positions, the intersection of the vectors is the answer. (right) An example of real data. The artificial shift of x,y = 0,± 2 pixels are shown. The position of the nine images corresponds to the shift. For example the top-left image is the difference of (x,y)=(-2,+2) shifted target image and the reference. The vectors show the direction toward the possible answer.

1, for example, it is clear that the target image is slightly shifted rightward compared with the reference image, since the positive pattern appears on the right of the negative pattern. The direction can be estimated by measuring the position of the positive and the negative peaks in the difference image. The next idea is to add artificial shifts to the subtracting image. The schematic figure and the real data are shown in Figure 2 left.

The estimation of the direction has some error. I therefore make 9 patterns, by adding artificial shifts by x,y = 0,± 2 pixels. In the example in Figure 2 right, the answer is to the right in the original position. If we add x=+2 shift to the target data, the answer is to the left. We therefore know that the best answer lies somewhere between x=0 and x=+2. The vectors intersects each other around the best solution. An example is shown in Figure 3.

I adopted the truncated least squares (Rousseeuw 1984) for the evaluation function for shift of (x,y), which is defined as

$$e(x,y) = \sum_{i=1}^{0.75N} d_i(x,y)^2,$$

where d_i is the distance between (x,y) and a vector sorted in the increasing order, and N is the number of the vectors. In this method, the vectors are 9, and $0.75N = 6$ vectors are used in the function for reducing the effect of outliers. The best position is estimated so that the e(x,y) is minimal.

4. Results

The method is implemented as a C-program. The artificial shift of the target image is implemented as a memory shift without any interpolation as the shifts are integer. A connected-pixels method (e.g. Bertin & Arnouts 1996) is used for

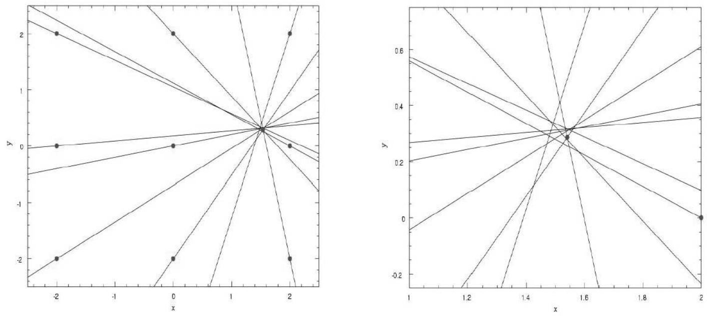

Figure 3. The estimation of the best position. (left) 9 shifted position and vectors. The dots represents the shifted position and the best position. (right) zoomed plot around the best position. The vectors do not intersect at a single position, because of errors.

the peak detection. The vector from the positive peak to neighboring negative peak is marked. The vector for the difference image is calculated as a median of the vectors. The C-program estimates the shift between two 1k×1k floating image in 0.5 sec with Intel Pen4 1.7GHz Linux machine. Then, a shift of the target image by a convolution of sinc function is performed for better difference image. An example is shown in Figure 4.

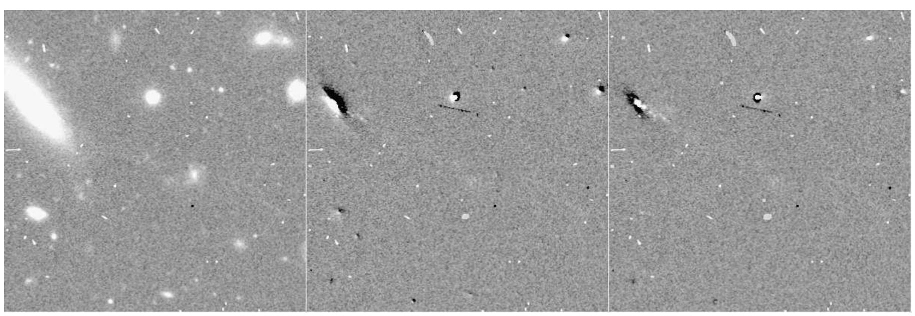

Figure 4. (left)R-band image of Coma cluster. (center)Difference of narrow-band image and the R-band image. Several positive-negative patterns exist because of a bad position alignment. (right) After the position refinement with this method. The positive-negative patterns almost disappear, and jet-like feature is now recognized from the galaxy at the left, which is a real narrow-band feature.

The difficulties of the current implementation are 1) the preprocess of flux matching of input images is required, and 2) the threshold of pattern detection should be set manually. These points should be solved in future version.

References

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